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EXAMPLE OF INTERACTION EFFECTS IN TRANSDUCER ARRAYS.(U)

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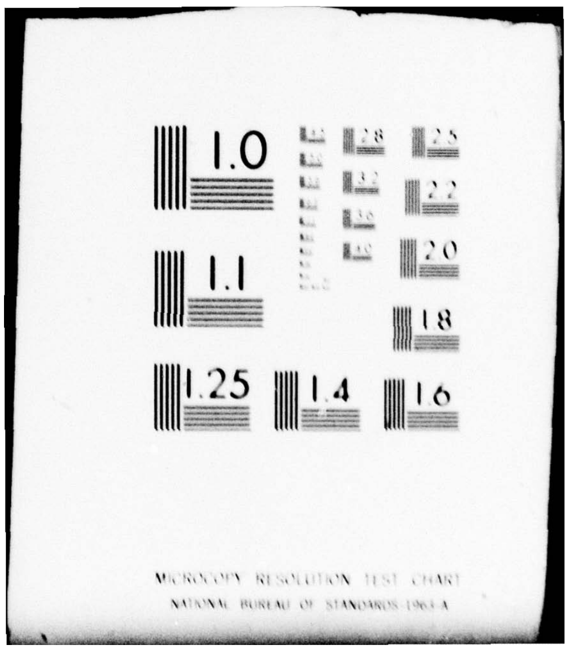
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USL Problem 963.2
No. 1-450-00-00
S-F001 03 04-8044

U. S. Navy Underwater Sound Laboratory
Fort Trumbull, New London, Connecticut

EXAMPLE OF INTERACTION EFFECTS IN TRANSDUCER ARRAYS.

by

Judith F. Atwood and Charles H. Sherman

USL Technical Memorandum No. 912-20-62

1 March 1962

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15p.

INTRODUCTION

There have been various calculations of acoustic interaction effects in transducer arrays ~~(for example, references (a), (b), (c))~~ and at least one experimental study specifically aimed at exploring such effects ~~(reference (d))~~. Of necessity, all this work considers special cases and provides little in the way of general guide lines which are useful for practical array problems. We shall present here some calculations which, although still in the special case category, cover a wide range of certain variables in an attempt to gain some insight into such problems. For example, the extent to which the transducer velocities in the array will vary when all transducers are driven by the same force is shown as a function of transducer spacing and individual transducer efficiency. The discussion is directed mainly at the case where the transducers are velocity limited, and the resulting limitations on radiated power are illustrated.

CALCULATIONS

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We have chosen an array consisting of one circular piston surrounded by six other pistons in an infinite rigid baffle (see Figure 1). The radiation resistance for an array with this geometry has been discussed by Pritchard (reference (a)) for the case where all pistons have the same velocity. Here we shall consider the case where all pistons are driven with the same internal force which more nearly corresponds to usual practice. We have calculated the piston velocities and radiation impedances in the array as a function of the distance (d) between pistons for various amounts of internal mechanical resistance in the transducers. We have specified a single value of ka, where a is the radius of the pistons, and a single value of the internal mechanical reactance of the transducers. We have used ka = 1/2 because the interaction effects of most interest are likely to occur when the individual pistons are considerably smaller than a wavelength. We have taken the internal mechanical reactance equal to the negative of the self radiation reactance. Thus, the results hold only at the frequency of velocity resonance of a single isolated transducer.

The center piston in this hexagonal array would be expected to show exaggerated interaction effects because its interaction with all the outer

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1. REPORT NUMBER	2. GOVT ACCESSION NO. 912-20-62	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EXAMPLE OF INTERACTION EFFECTS IN TRANSDUCER ARRAYS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER 1-450-00-00
7. AUTHOR(s) Atwood, Judith F. and Sherman, Charles H.		8. CONTRACT OR GRANT NUMBER(s) Nonr266(84)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Underwater Systems Center New London, CT		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research, Code 220 800 North Quincy St. Arlington, VA 22217		12. REPORT DATE 1 MAR 62
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
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pistons is the same, and the possibility of a large net interaction exists. In this respect the present array is similar to Rusby's cross-shaped array (reference (d)) and would be expected to show the kind of effects which he reported. The interaction effects found here are probably greater than would be found in most other arrays where more cancellation of interactions usually occurs.

The equations of motion of the center piston and any one of the outer pistons in the array are taken to be

$$(Z_M + Z_1) U_1 e^{i\mu_1} = F, \tag{1}$$

$$(Z_M + Z_2) U_2 e^{i\mu_2} = F, \tag{2}$$

where subscript 1 refers to the center piston and subscript 2 to the outer piston. Z_M is the internal mechanical impedance which we take to be

$$Z_M = R_M + i X_M = R_M - i X_{11} \tag{3}$$

where X_{11} is the self radiation reactance ($Z_{11} = R_{11} + i X_{11} = (.1199 + .3969i)\rho c A$ for ka = 1/2) Z_1 and Z_2 are the radiation impedances of the center and outer pistons which in this case (see Figure 1) are given by

$$Z_1 = Z_{11} + 6 Z_{12} \frac{U_2}{U_1} e^{i(\mu_2 - \mu_1)} \tag{4}$$

$$Z_2 = Z_{11} + 2 Z_{12} + 2 Z_{24} + Z_{25} + Z_{12} \frac{U_1}{U_2} e^{i(\mu_1 - \mu_2)} \tag{5}$$

where the Z_{12}, etc. are mutual impedances which have been computed for this case by Barrett (reference (e)) from the equation derived by Pritchard (reference (a)). $U_1 e^{i\mu_1}$ and $U_2 e^{i\mu_2}$ are the two velocities and F is the magnitude of the internal driving force. Since the driving force is the same for all pistons in the array, we use it as the phase reference.

When Eq. (4) and Eq. (5) are substituted into Eq. (1) and Eq. (2) the latter two equations can be solved for the two velocities in terms of the common driving force F. It seems more instructive, however, to express the velocity magnitudes in terms of the velocity magnitude of a single, isolated piston driven by the same force F. For this latter velocity magnitude we find, using Eq. (3)

$$U_{11} = \frac{F}{R_M + R_{11}} \tag{6}$$

Figures 2 and 3 show the two velocity ratios U_1/U_{11} and U_2/U_{11} as functions of $d/2a$ for R_M/R_{11} = 1.00, 0.43 and 0.11 which correspond to single transducer mechanoacoustical efficiencies ($\frac{R_{11}}{R_M + R_{11}}$) of 50%, 70% and 90%, respectively. The phase difference between the two velocities in the array, $\mu_1 - \mu_2$ and the velocity amplitude ratio U_1/U_2 are given in Figure 4. From the velocity magnitudes and phases the two radiation impedances can be found from Eq. (4) and Eq. (5). The results are shown in Figures 5 and 6, which also show radiation resistance curves for equal piston velocities. The calculations have been done for values of d from closest possible packing ($d/2a$ = 1) to quite loose packing ($d/2a$ = 2.36).

DISCUSSION

Figures 2 and 3 show that for tight packing of the array the velocity amplitudes are smaller than for a single piston driven by the same force. This occurs because the radiation impedance is higher in the array than it is for a single piston. As the packing becomes looser the velocity of the outer pistons increases, which would be the most common behaviour in any array. However, the possibility of quite different behaviour is shown by the center piston where the velocity may increase or decrease, depending on the value of R_M. Note especially that for small R_M the velocity of the center piston may be significantly greater than it would be for a piston operating alone and driven by the same force. Such an effect could lead to the mechanical failure of some of the transducers in an array if the transducer velocity limitations did not contain a considerable safety factor. Although the behaviour shown by the center piston is exaggerated by the type of array under study, it is likely that in any large array the environment of one or more of the pistons would be such as to cause similar effects.

Other calculations of U_1/U_{11} were made at $d/2a$ = 1.96 for $R_M > R_{11}$, and they show, as expected, that U_1/U_{11} approaches unity as R_M/R_{11} increases. It was also found that at $d/2a$ = 1.96 the ratio U_1/U_{11} is a minimum for $R_M/R_{11} \approx 0.96$. Thus, the curve for R_M/R_{11} = 1 shows approximately the extreme behaviour.

Note by comparing Figure 2 and Figure 4 that the velocity peak of the center piston for the smallest R_M occurs when the center and outer velocities are nearly 180° out of phase. It should also be pointed out that the velocity ratios in Figure 2 and Figure 3 pass through other oscillations before approaching unity at large d. For example, the ratio U_1/U_{11} has a peak near kd = $3\pi/2$ where the mutual radiation resistance between the center and outer pistons has its greatest negative value.

Figure 5 shows that the radiation resistance of the center piston becomes negative and remains negative over a considerable range of spacing. This means that the center piston is absorbing some of the acoustic energy radiated by the outer pistons. Rusby observed the same kind of effect in his measurements. It should be noted that the velocity peak of the center

piston does not occur when $R_M + R_1 = 0$, because the transducers in the array are not exactly at resonance. This peak does occur when $|Z_M + Z_1|$ is a minimum, as can be seen from Eq. (1).

It is of interest to note that the quantity $|Z_M + Z_j|$ is never zero for any piston of any array. When the set of linear equations relating the piston velocities to the driving forces for any array with any set of driving forces are solved by determinants, we have for any one of the velocities

$$u_j = \Delta_j / \Delta \tag{7}$$

where Δ is the determinant of the coefficients and Δ_j is Δ with the j^{th} column replaced by the set of driving forces. The determinant Δ is never zero. If it were, there would be a solution other than all velocities equal to zero for the case where all the driving forces were zero, and this is physically impossible. Since Δ is never zero none of the velocities are ever infinite. It follows that the quantity $|Z_M + Z_j|$ for any piston in any array is never zero. Both $R_M + R_j$ and $X_M + X_j$ may be zero, but they are never zero under the same conditions.

In Figure 6 the radiation resistance curve for equal piston velocities is a member of the family of curves shown there corresponding to $R_M >> R_{11}$, since large R_M and equal driving forces result in nearly equal velocities. This is fairly obvious in Figure 6. It is not obvious in Figure 5, but must also be true there.

The total acoustic power radiated by the array can be written

$$P_A = \frac{1}{2} R_1 U_1^2 + 3 R_2 U_2^2 \tag{8}$$

We have calculated and plotted on Figure 7 the quantity

$$\frac{P_A}{\frac{1}{2} R_{11} U_{max}^2} = \begin{cases} \dfrac{R_1}{R_{11}} + 6 \dfrac{R_2}{R_{11}} \left(\dfrac{U_2}{U_1}\right)^2 & \text{for } U_1 > U_2 , \\[2ex] \dfrac{R_1}{R_{11}} \left(\dfrac{U_1}{U_2}\right)^2 + 6 \dfrac{R_2}{R_{11}} & \text{for } U_2 > U_1 . \end{cases} \tag{9}$$

This is the ratio of the total power radiated by the array to the power which would be radiated by a single isolated piston vibrating with a velocity equal to the higher of the two different piston velocities in the array. This is the power ratio which is pertinent to the case of velocity limited transducers. The curves in Figure 7 show the power which could be radiated by this array if the higher of the two piston velocities in the array were limited to the value U_{max}.

For the lower values of R_M the power radiated by the seven piston array is less than seven times the power radiated by one piston. This occurs mainly because the velocities of the six outer pistons must be kept lower than the limiting value in order to prevent the center piston velocity from exceeding this value. In the interest of radiating power it would be better to remove the center piston. However, for higher R_M (for example, $R_M = R_{11}$) the situation is quite different. The outer piston velocity usually exceeds the inner piston velocity, and the power radiated by the array is more than seven times that radiated by one piston. The large negative radiation resistance of the center piston for $R_M = R_{11}$ has little effect on the power because it is accompanied by a very low velocity.

Figure 7 also shows the power which would be radiated by the array if all pistons had the same velocity. Again this curve is a member of the family of curves in Figure 7 for $R_M >> R_{11}$. It is clear that the power radiating capability of the array for a given maximum piston velocity improves as the equal velocity condition is approached. However, for $R_M = R_{11}$ we see that at $d/2a \approx 1.5$ the power comes close to that in the equal velocity case. It can be seen from Figure 4 that this spacing corresponds to nearly equal velocity magnitudes, although the two velocities are about 100° out of phase.

If the object is to radiate as much power as possible from a given array, there is sometimes then an advantage in using transducers which are not highly efficient. This is done at the cost of lowering the efficiency of the array, although not to a great extent, as is shown by Figure 8, which gives the mechanoacoustical efficiency of the array defined by

$$\eta_A = \frac{P_A}{P_A + (\frac{1}{2}u_1^2 + 3u_2^2)R_M} \tag{10}$$

The desirability of having approximately equal piston velocities in arrays of small pistons intended for the radiation of high power has become fairly clear. The equal velocity condition could be achieved by providing each transducer with its own suitably adjusted driver. Or, it might be achieved by the use of different transducers in different locations in the array. Another crude approach is the use of transducers which, when operating alone, are relatively inefficient. This approach may not be as bad as it seems since Figure 8 shows that the array efficiency may be considerably higher than the individual transducer efficiency if the latter is not too high.

The single frequency at which these calculations have been made is a serious limitation on their usefulness. We hope, therefore, to extend this work by repeating some of the calculations at frequencies which differ by small amounts from the resonant frequency of a single, isolated transducer.

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Mathematician

Charles H. Sherman
CHARLES H. SHERMAN
Physicist

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(b) R. V. Baier, "Evaluation of Effects of Element Interactions in a Large Low-Frequency Array", NRL REPORT No. 5251, February 1959 (CONFIDENTIAL) (See also Report No. 5355 and 5491 on same subject.)

(c) C. H. Sherman and D. F. Kass, "Radiation Impedances, Radiation Patterns and Efficiency for a Large Array on A Sphere", USL RESEARCH REPORT No. 429, July 1959

(d) J. S. M. Rusby, "Investigation of a Radiation Impedance Anomaly Between Sound Projectors Mounted in an Array", U. S. NAVY JOURNAL OF UNDERWATER ACOUSTICS, vol. 11, p. 463, (July 1961) (CONFIDENTIAL)

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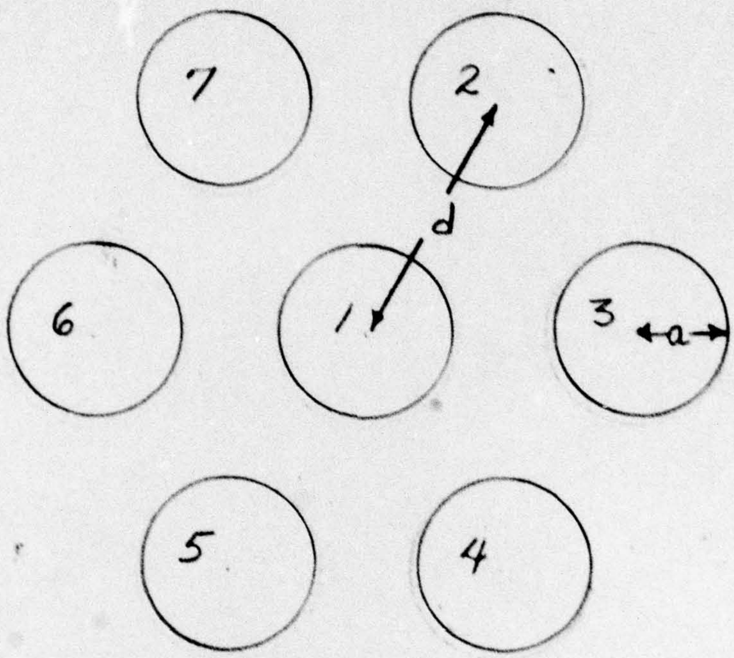

Figure 1. Geometry of the Array

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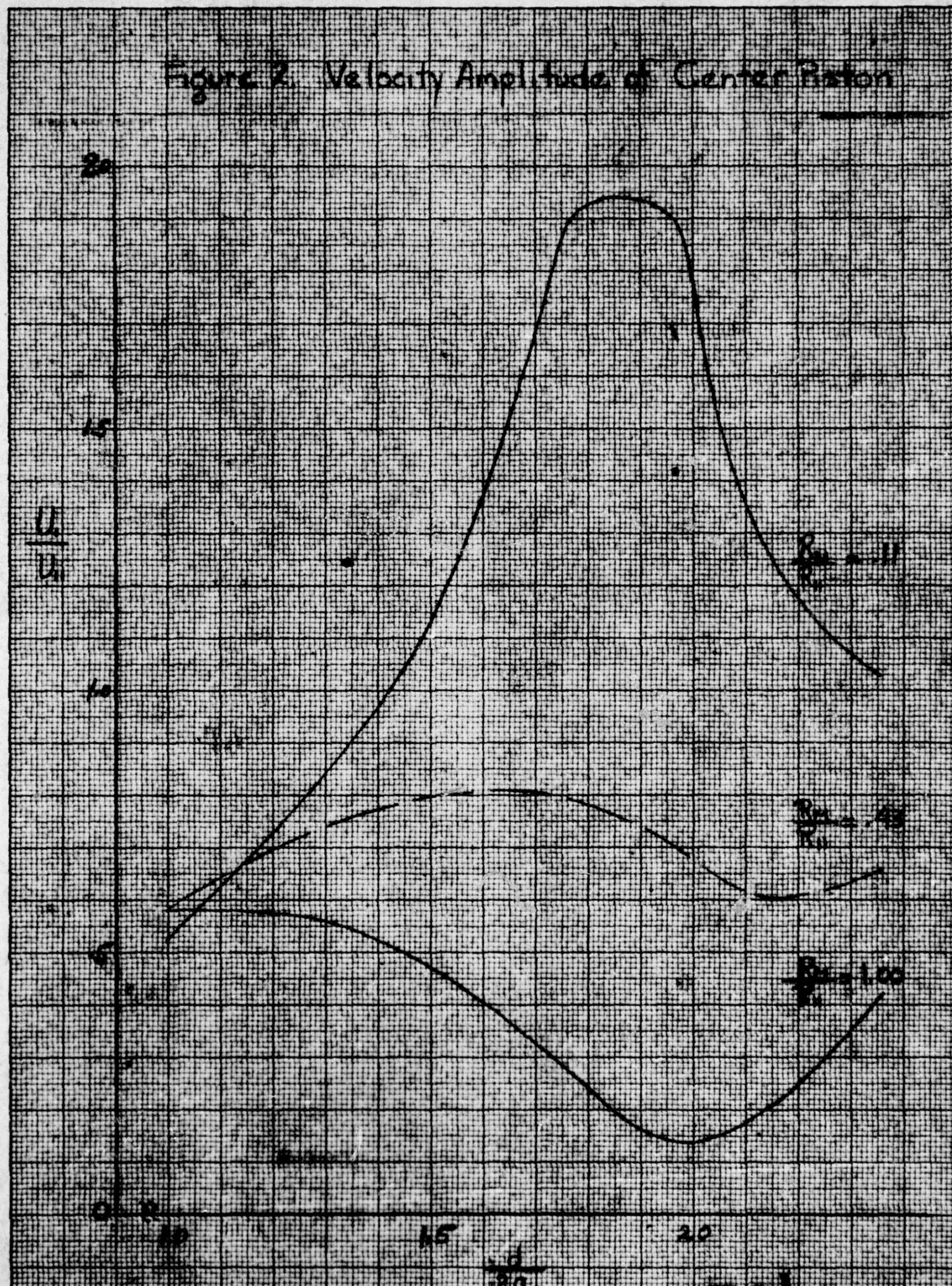
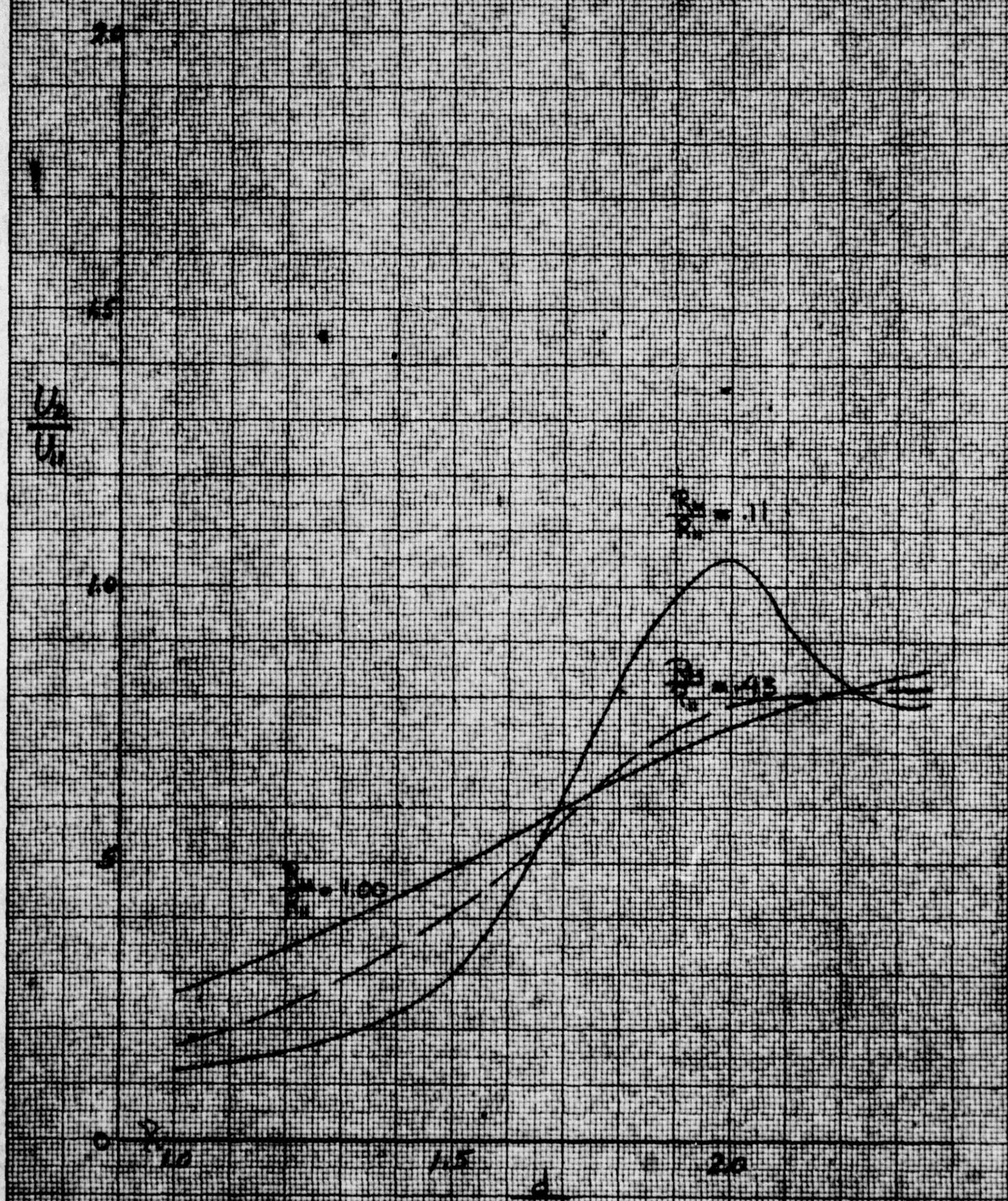


Figure 2. Velocity Amplitude of Center Piston
2.0
1.5
1.0
0
1.0
1.5
2.0
d/2a
= .11
= .43
= 1.00

Figure 3. Velocity Amplitude of Outer Piston



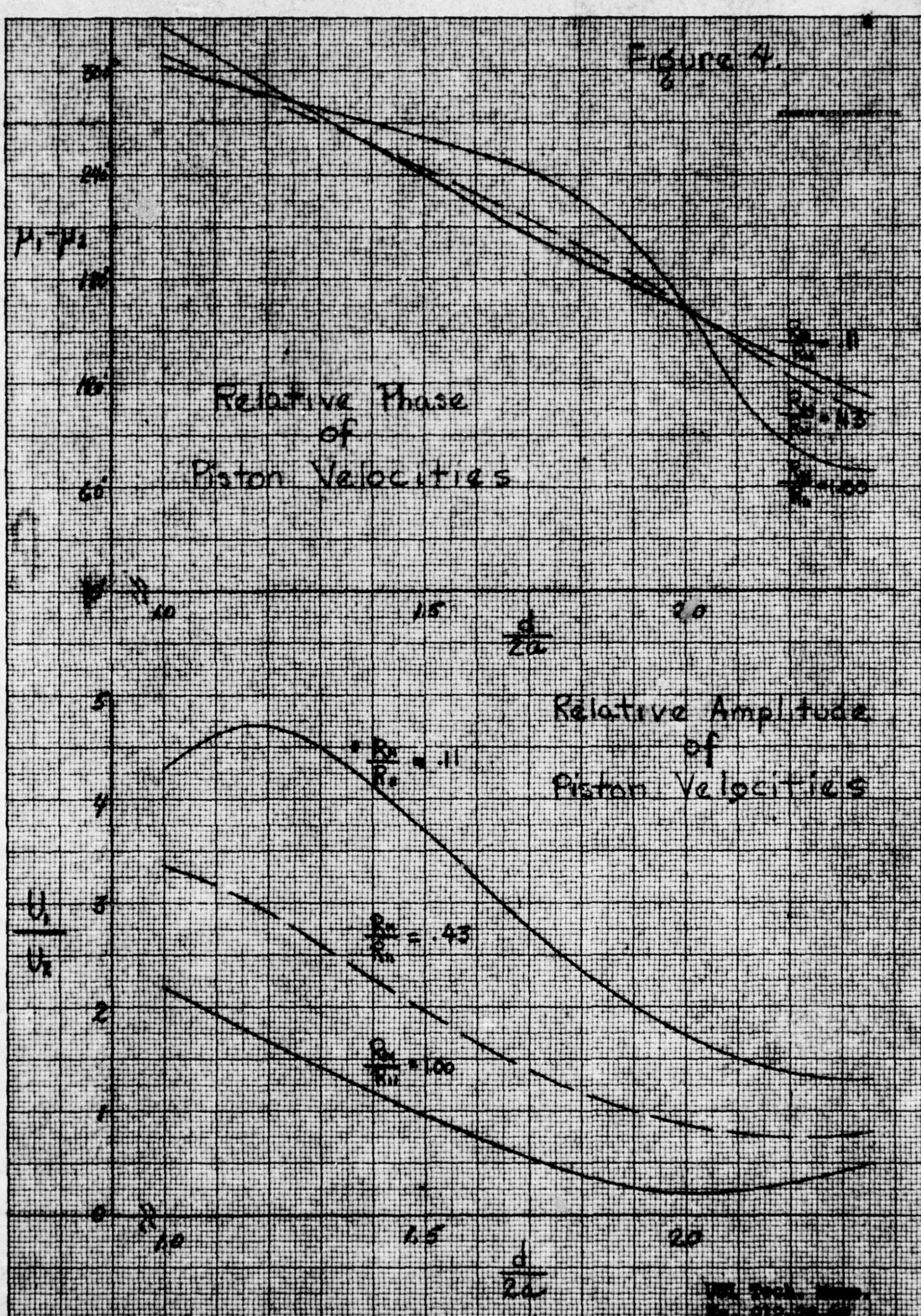
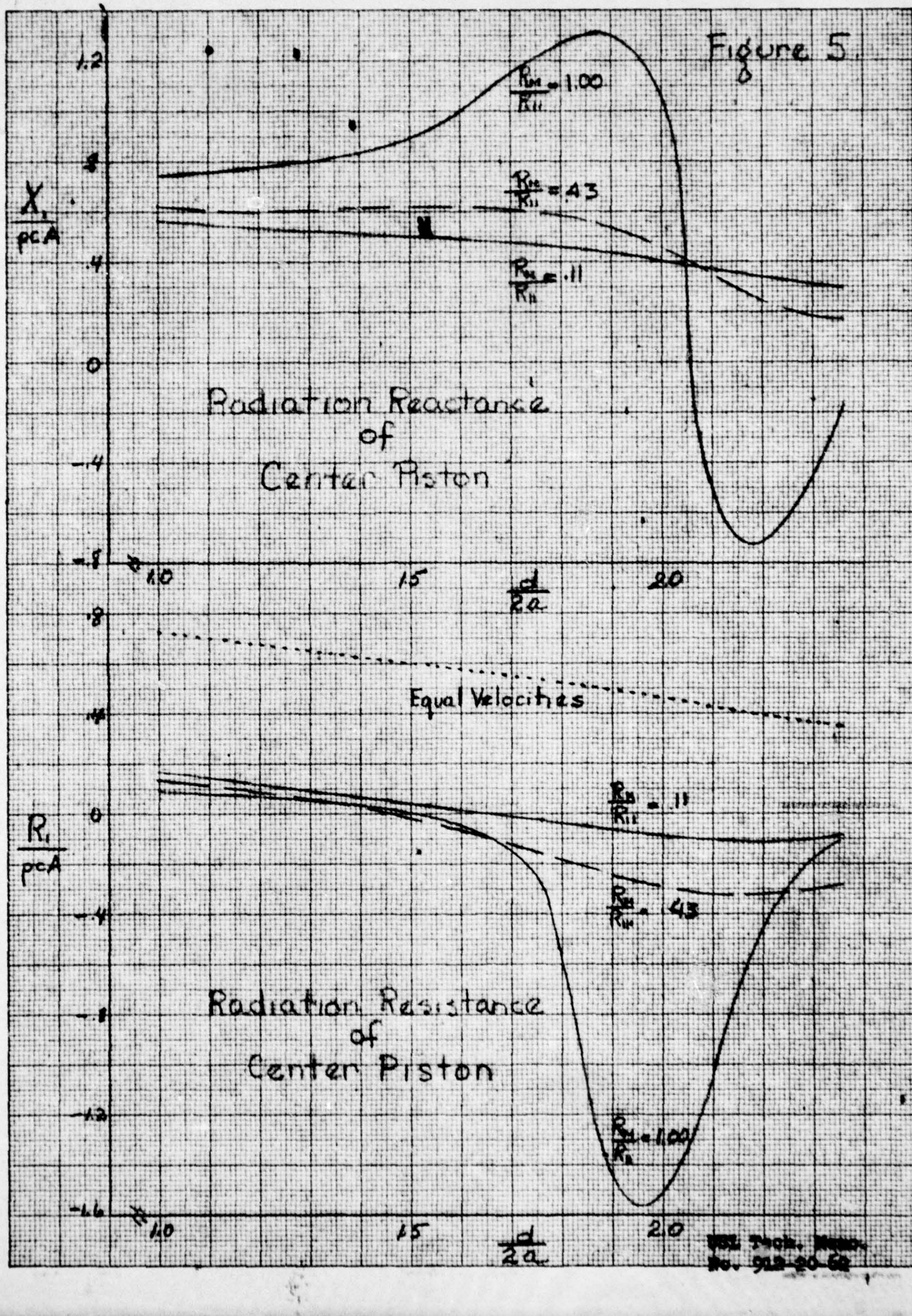


Figure 4.
Relative Phase of Piston Velocities
$\mu_1 - \mu_2$
Relative Amplitude of Piston Velocities
$\frac{U_1}{U_2}$
$\frac{d}{2a}$
1.0
1.5
2.0
$\frac{R_{21}}{R_{11}} = .11$
$\frac{R_{21}}{R_{11}} = .43$
$\frac{R_{21}}{R_{11}} = 1.00$



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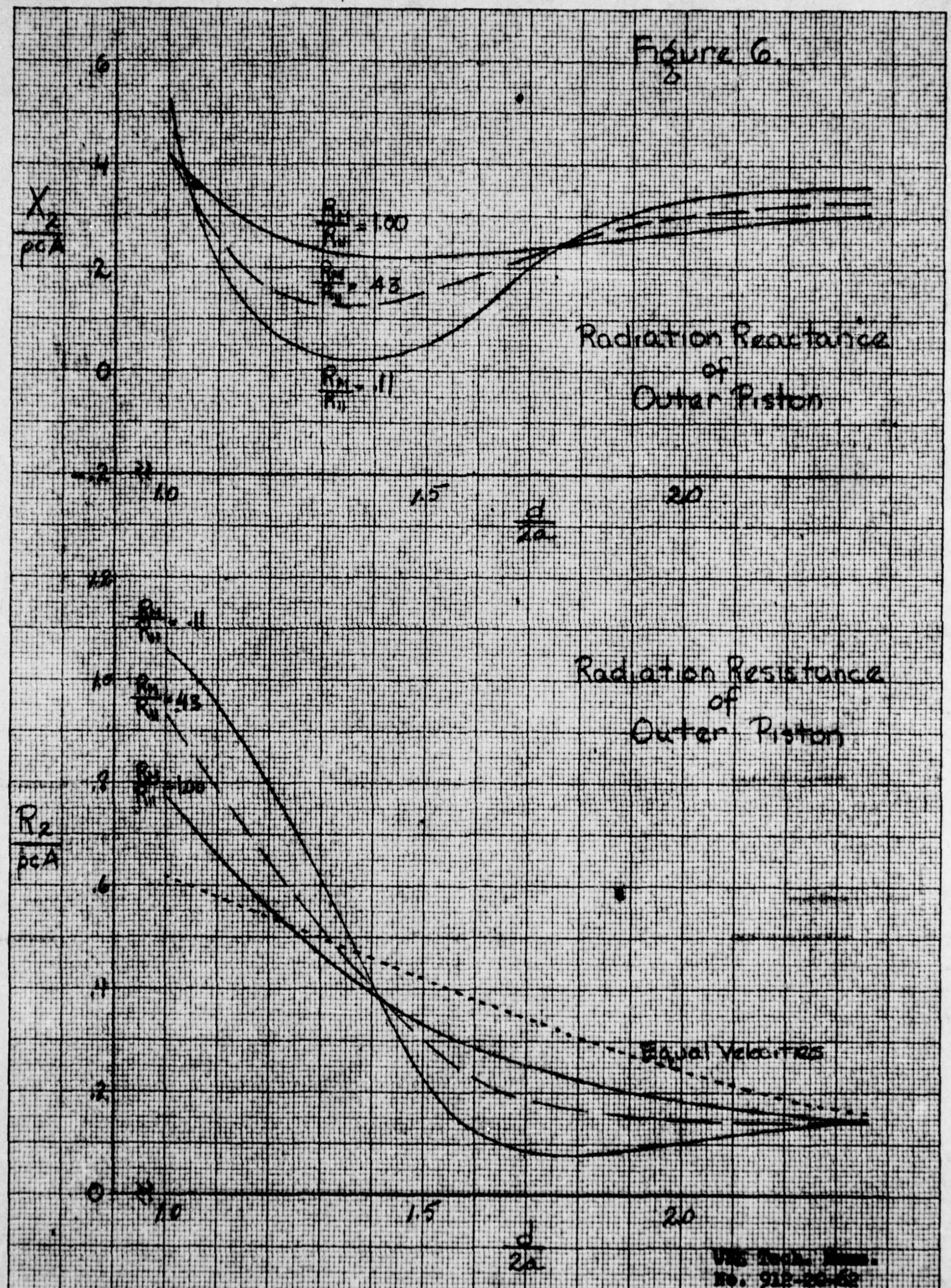

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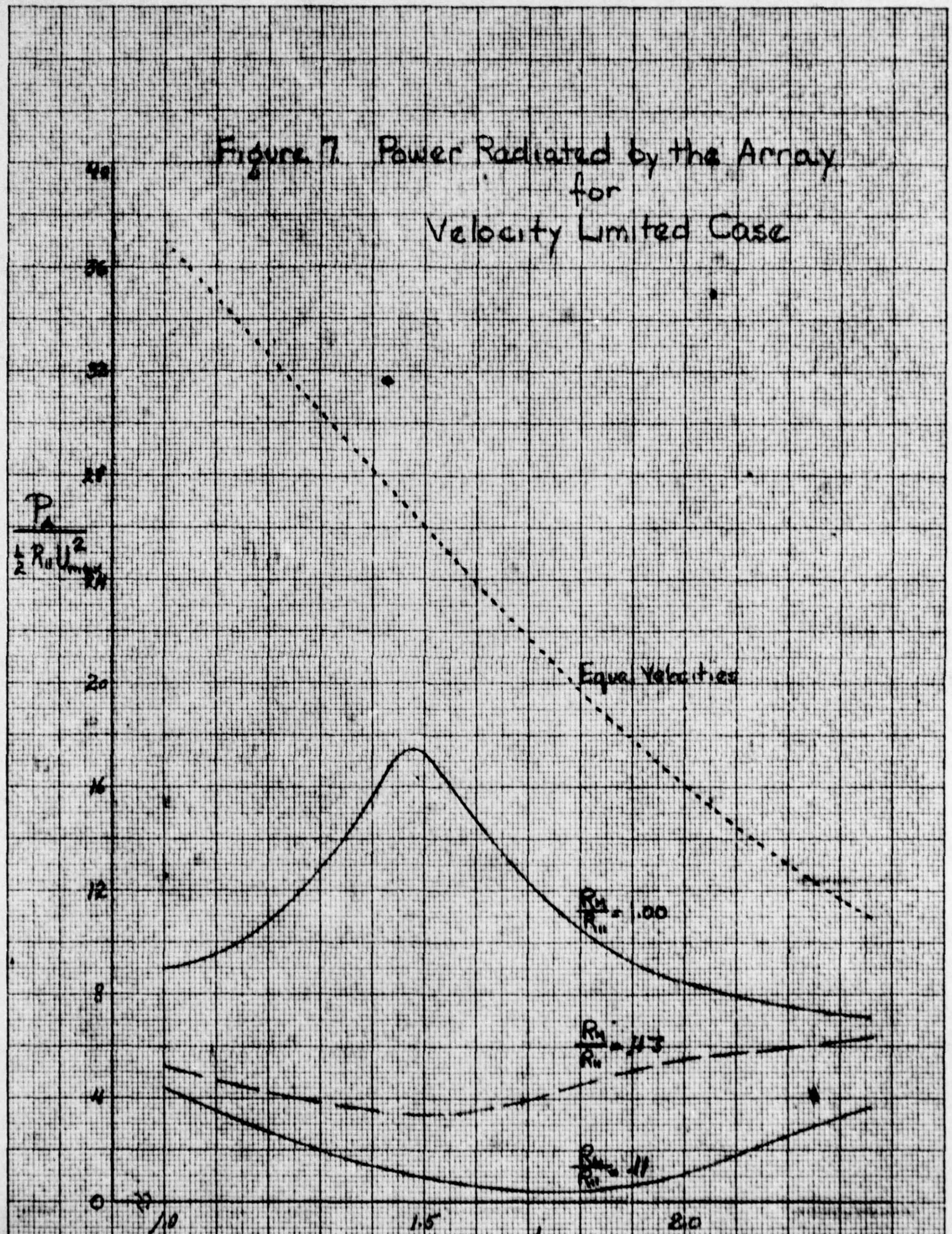

Figure 7. Power Radiated by the Array for Velocity Limited Case

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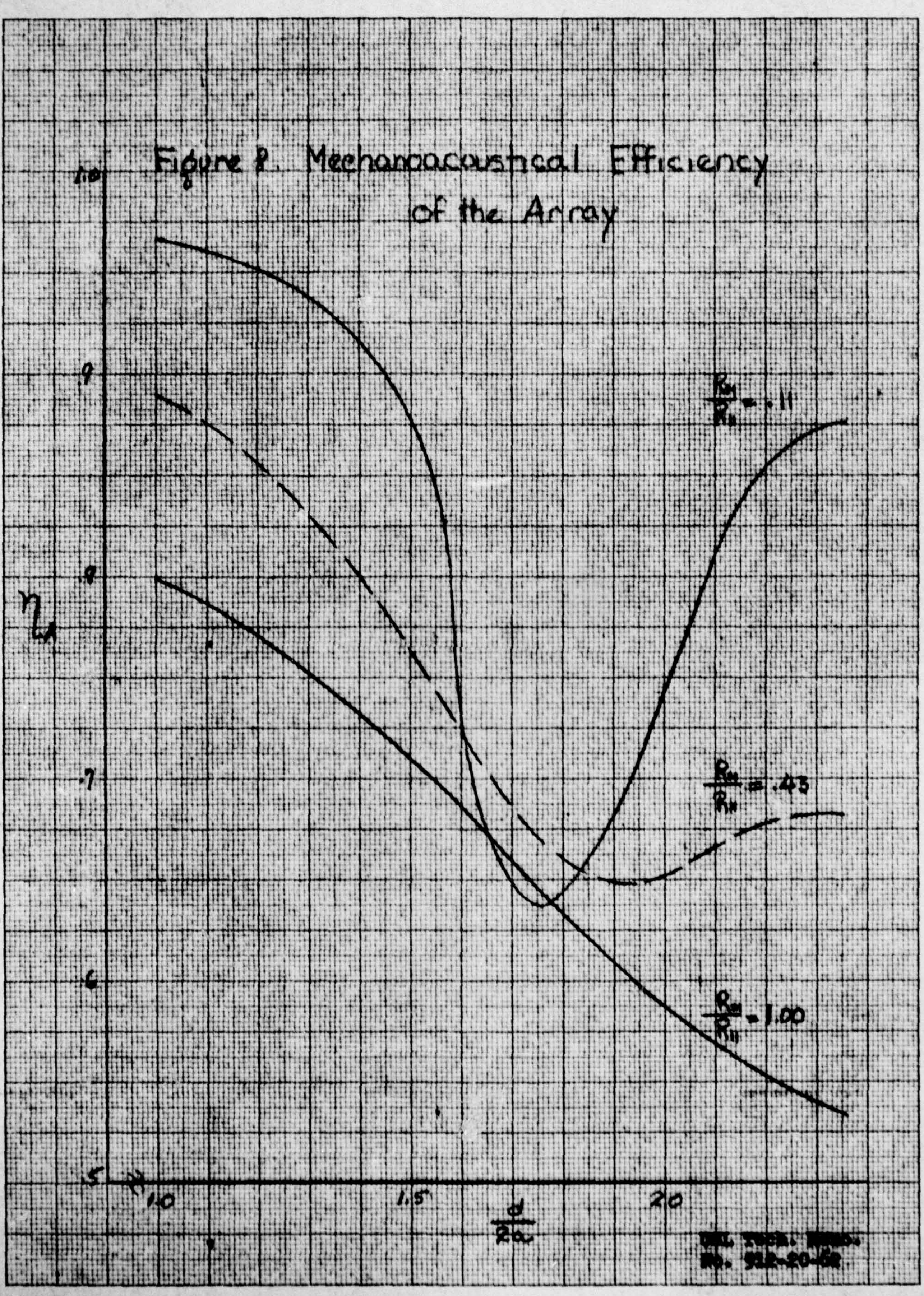

Figure 8. Mechanoacoustical Efficiency of the Array
1.0
.9
.8
.7
.6
.5
η_A
1.0
1.5
2.0
$\frac{d}{2a}$
$\frac{R_{12}}{R_{11}} = .11$
$\frac{R_{12}}{R_{11}} = .43$
$\frac{R_{12}}{R_{11}} = 1.00$